Estimation of a Sensitivity-Based Metric for Detecting Market Power

HyungSeon Oh, M.ASCE1; and Robert J. Thomas2

Abstract: The abuse of market power is a potentially serious problem for market designers. Few indices, if any, exist to measure the potential for market power in real time. In this regard, Murillo-Sanchez et al. derived an expression for a dispatch-to-price sensitivity matrix, M. The expression requires information about network topology and parameters, as well as the rules used to operate the market. While computing the matrix is conceptually easy for those with all the market and system information, such as an independent system operator, the method is probably impractical for market participants due to the inaccessibility of much of the information. In this paper, a method for estimating the M matrix by using publically available data are suggested, indicating that any market participant who does the computation will know when conditions permit them to lower or raise prices through decreased/increased bids and/or offers.

DOI: 10.1061/(ASCE)EY.1943-7897.0000019

CE Database subject headings: Electric power; Pricing; Electric transmission; Sensitivity analysis; Matrix methods; Computation.

Author keywords: Dispatch sensitivity matrix; Market power; Null space; Kuhn-Tucker optimality conditions; Power transfer distribution factor (PTDF) matrix.

Introduction

If a market is competitive, the electric power generated by others can substitute for the electric power provided by any generator. However, it is possible that a generator or subset of generators will have very low substitutability because of their location in a network. When this happens and generators exploit the situation, their prices can increase without an attendant reduction in their quantity dispatched. As the dispatch sensitivity with respect to the price approaches zero, the dispatch is affected less and less by a change in price. In other words, the corresponding generators can increase their price without sacrificing their dispatch, and the potential revenue for such generators will increase accordingly.

A few standard metrics such as the hedge ratio Herfindahl-Hirschman index and the Lerner index have been used in an attempt to measure market power. While these indices are useful in the settings for which they were developed, they are not suitable for real-time market monitoring (Kian et al. 2004; Lesieutre et al. 2003; Murillo-Sanchez et al. 2001). Furthermore, they do not account explicitly for transmission network effects. Several other indices, such as the pivotal supplier index (Bushnell et al. 1993), the residual supply index (Rahimi and Sheffi 2003), the must-run-ratio (Gan and Boureir 2002a,b), and the must-run share (Wang et al. 2004), have been proposed to take locational market power into account by considering the network.

Murillo-Sanchez et al. presented a mathematical derivation for the dispatch-to-price sensitivity matrix M (Murillo-Sanchez et al. 2001). By using this M matrix approach, it is possible to identify subsets of generators that have market power (Bernard et al. 1998; Kian et al. 2004; Lesieutre et al. 2003; Murillo-Sanchez et al. 2001; Schulze et al. 2000; Zimmerman et al. 1999). While the method is practical for ISO’s who have all of the information necessary to compute the M matrix, it is impractical for market participants, such as generators, because they do not have access to the same level of information. However, if the actual M matrix can be estimated with good precision by using only publically available data, then market power for each participant or groups of participants can be assessed by all those interested in market functions.

In this paper, a method for estimating the M matrix is presented. Using the technique, M matrices for several operating conditions of a modified IEEE 30-bus system are estimated and compared to the true M matrices.

Dispatch Sensitivity Matrix

Currently, those participating in real-time markets for energy get paid real power prices. For this case, and when interconnecting network effects are not considered, the price paid for real power is the sensitivity of the total system cost, cost(\(x\)), with respect to dispatch. Consequently, the cost is reduced to cost(\(q, \rho\)), the total system cost for the vector of offers \(q\) and \(\lambda\) submitted by generators. A full alternating current (ac) optimal power flow (OPF) formulation and the corresponding Lagrangian are given by

\[
\min_{x} \text{cost}(x)
\]

subject to

\[
F(x) = 0 \quad \text{and} \quad G(x) \leq 0
\]
\[ \begin{align*}
\rightarrow & \min_\mathbf{z} \quad \text{cost}(\mathbf{z}) + \eta_1^T \mathbf{F}(\mathbf{z}) + \eta_0^T \mathbf{G}(\mathbf{z}) \\
\text{s.t.} & \quad \mathbf{M} \mathbf{z} = \mathbf{P}_d - \mathbf{P}_{loss}, \quad \mathbf{z} \geq 0
\end{align*} \] (1)

Then the optimal levels of generation that minimize cost while satisfying the constraints \( \mathbf{M} \mathbf{z} = \mathbf{P}_d - \mathbf{P}_{loss} \), for some appropriate choice of the scalar matrix \( \mathbf{P}_{loss} \), can be written as \( \mathbf{z}(\mathbf{g}^*, \rho^*) = \mathbf{F}(\mathbf{g}^*, \rho^*) = \mathbf{z}^* \). The \( \mathbf{M} \) matrix is, by definition, the sensitivity of dispatch to price. With \( \mathbf{g}^* \) and \( \rho^* \), we have

\[ \mathbf{M} = \nabla_{\mathbf{g}, \rho} \text{cost}(\mathbf{g}^*, \rho^*) = \nabla_{\mathbf{g}, \rho} \text{cost}(\mathbf{g}^*, \rho^*) \]

or

\[ \Delta \mathbf{g}^* = \mathbf{M} \Delta \rho^* \] (2)

The \( \mathbf{M} \) matrix is the second derivative of cost with respect to price and is therefore symmetric. Because \( \mathbf{g}^* \) and \( \rho^* \) are optimizers that minimize the objective function, the \( \mathbf{M} \) matrix must be negative semidefinite. Symmetry can also be observed from the equation for \( \mathbf{M} \) (Murillo-Sanchez et al. 2001). When an eigenvector of a matrix is multiplied to the matrix, the resulting vector is parallel to the eigenvector with the magnitude of the corresponding eigenvalue. Therefore, any change in price parallel to an eigenvector of \( \mathbf{M} \) yields a dispatch change in the same direction of the price change. Because \( \mathbf{M} \) is negative semidefinite, the increase in price results in nonpositive change in dispatch. For the price change in the space formed by the eigenvectors of which corresponding eigenvalues are zero, there is no change in dispatch. The most striking example of such a price change is a load pocket—a situation where a deficiency in transmission capacity to a market area cannot be priced away sufficiently to clear the market during peak-load periods.

The original ac OPF shown in Eq. (1) yields the dispatch-to-price sensitivity in matrix \( \mathbf{M} \) (Murillo-Sanchez et al. 2001). However, evaluating the matrix requires confidential data. In this study, a two-step process was developed to approximate the original OPF problem to: (1) solve the original ac OPF problem described in Eq. (1), the first OPF, and (2) create a reasonable economic dispatch problem that includes the network, where the second OPF has the same solution as the first. The second step requires solving the second OPF, creating a reasonable approximation to the \( \mathbf{M} \) matrix. In this second OPF, all parameters are assumed to be fixed. Multiple generators located at a single bus are treated as a single generator in the second OPF. All the offers submitted by generators are replaced by \( \rho^* \); therefore, it is not possible to construct the second OPF without performing the first OPF. This setup allows the sensitivity of real power injection with respect to \( \rho^* \) to be evaluated. A discussion of the degenerate case of a lossless system appears in the Appendix. If the offer submitted by generators is \( \rho^* \), all generators are partially dispatched. As a result, the following equivalent economic dispatch problem (second OPF) can be constructed

\[ \begin{align*}
\min_{\mathbf{q}} & \quad \rho^T \mathbf{q} \\
\text{s.t.} & \quad 1^T (\mathbf{q} - \mathbf{c}) = \mathbf{P}_{loss}(\mathbf{q} - \mathbf{c}) + \mathbf{P}_D \\
& \quad 1^T \mathbf{q} = \mathbf{P}_{loss}(\mathbf{q}) + \mathbf{P}_D
\end{align*} \]

where

\[ \begin{align*}
\mathbf{P}_D = \mathbf{P}_D^* + 1^T \mathbf{d} \\
\mathbf{H}(\mathbf{q} - \mathbf{c}) = \mathbf{c}' - \mathbf{H} \mathbf{c} = \mathbf{c}' + \mathbf{H} \mathbf{c} = \mathbf{c}
\end{align*} \] (3)

where \( \mathbf{P}_{loss} \) and \( \mathbf{P}_{loss} \) stand for total transmission loss as functions of net injection \( (\mathbf{q} - \mathbf{c}) \) and injection \( \mathbf{q} \), respectively. It is worthwhile to note that losses play an important role in the dispatch sensitivity matrix because price setters are not differentiable in their offer prices without losses. Suppose some units exist where bus injections are fixed contractually, because of market mitigation policies, or for some other reason. These units are termed "must-run" units. Then, "must-run" generating units are excluded from Eq. (3) by treating them as negative inelastic loads.

In Eq. (3), inequality constraints associated with generation capacity are incorporated with \( \rho^T \) because \( \lambda^T \) and associated multiplier sum up to \( \rho^T \). Other inequality constraints such as voltage constraints and reactive power generation capacity constraints are not considered in Eq. (3).

Real power injections from demand and "must-run" units are included in the vector \( \mathbf{d} \), which is fixed throughout this study. Each column of \( \mathbf{H} \) is obtained from the appropriate columns of the power transfer distribution factor (PTDF) matrix corresponding to generation buses so that it relates generation power injection to line flow. The cardinality of matrix \( \mathbf{H} \) is \( K \times N_q \). In this problem, \( \rho^* \) is obtained from the first OPF while leaving the generation dispatches variables, and real power losses in transmission networks are modeled in Eq. (3). Eq. (3) can be solved by forming the Lagrangian as follows:

\[ \begin{align*}
\Psi = \rho^T \mathbf{q} + \mu (\mathbf{P}_D - \mathbf{P}_{loss} - 1^T \mathbf{q}) + \sigma (\mathbf{c} - \mathbf{E} \mathbf{q})
\end{align*} \] (4)

where \( \mathbf{c} = \mathbf{c} \) defined in Eq. (3) of congested lines.

Because Eq. (4) is homogeneous in \( \rho^* \) and of degree one (Murillo-Sanchez et al. 2001), \( \Delta \rho^* \) in the direction of \( \rho^* \) will not alter generation dispatch. Therefore, \( \mathbf{M} \) has one structural eigenvector whose associated eigenvalue is zero. That is

\[ \Delta \mathbf{g}^* = \mathbf{M} \Delta \rho^* |_{\Delta \rho^* = 0} = \mathbf{c} \rho^* = 0 \] (5)

where \( \mathbf{c} \) = nonzero scalar. The Kuhn-Tucker optimality conditions are

\[ \begin{align*}
f &= \nabla_{\mathbf{g}} \Psi = \rho^* - \mu^* (1 - \nabla_{\mathbf{g}} \mathbf{P}_{loss}) - E^T \sigma = 0 \\
h &= \nabla_{\mathbf{q}} \Psi = 1^T \mathbf{g}^* - \mathbf{P}_D - \mathbf{P}_{loss}(\mathbf{g}^*) = 0 \\
f l &= \nabla_{\mathbf{q}} \Psi = - \mathbf{E} \mathbf{g}^* = 0
\end{align*} \] (6)

Suppose the price is disrupted from the \( \rho^* \). In this case, the real power injections and the shadow prices (\( \mu \) and \( \sigma \)) will also change. However, the subset of active constraints will not change under the infinitesimal price perturbation. The optimality conditions in terms of the perturbation about the original solution are then

\[ \Delta f = \Delta \rho^* + \mu^* (\nabla_{\mathbf{g}} \mathbf{P}_{loss}) \Delta \mathbf{g}^* - \Delta \mu (1 - \nabla_{\mathbf{g}} \mathbf{P}_{loss}) - E^T \Delta \sigma = 0 \]

\[ \rightarrow 1 - \nabla_{\mathbf{g}} \mathbf{P}_{loss} = \frac{1}{\Delta \mu} (\Delta \rho^* + \mu^* (\nabla_{\mathbf{g}} \mathbf{P}_{loss}) \Delta \mathbf{g}^*) - \frac{1}{\Delta \mu} E^T \Delta \sigma \] (7)

\[ \Delta h = (1 - \nabla_{\mathbf{g}} \mathbf{P}_{loss})^T \Delta \mathbf{q} = 0 \] (8)

\[ \Delta f l = \mathbf{E} (\mathbf{g}^* + \Delta \mathbf{g}^*) - \mathbf{E} \mathbf{g}^* = 0 \] (9)

One can combine Eqs. (7) and (8) to find Eq. (10a). Then the second term vanishes by using Eq. (9), and Eq. (10b) can be obtained from the definition of \( \mathbf{M} \) matrix shown in Eq. (1)

\[ \frac{1}{\Delta \mu} (\Delta \rho^* + \mu^* (\nabla_{\mathbf{g}} \mathbf{P}_{loss}) \Delta \mathbf{g}^*) - \frac{1}{\Delta \mu} E \Delta \mathbf{g}^* = 0 \]

(10a)
\[ \Delta \rho^* (I + \mu^* (V_{g} P_{\text{loss}}) M^T) \Delta \rho^* = 0 \]  

(10b)

Eq. (10b) should be satisfied for all \( \Delta \rho^* \). Consequently, the following can be written

\[ (B^{-1} + \mu^* M)^T B^T M = A^T B^T M = 0 \iff \text{M(BA)} = 0 \]

where \( A = B^{-1} + \mu^* M \) 

(11)

Note that the B matrix is symmetric because B is the second derivative of system loss function with respect to generation. Consequently, \( B^{-1} \) and A are symmetric. One finds

\[ \Delta \rho^* = [B^{-1} + \mu^* M] \Delta \rho^* = B^{-1} \mu^* M \Delta \rho^* = B^{-1} \rho^* \]

(12)

because \( \rho^* \) is in the null space of \( M \). Eq. (9) yields

\[ E \Delta g^* = E M \Delta \rho^* = 0 \iff E M = 0 \]

(13)

The right side of the arrow in Eq. (13) holds because the left side of the arrow is true for all perturbations of \( \rho^* \).

From Eqs. (5) and (13), the null space of \( M \) is spanned by \( \rho^* \), plus the row vectors of E matrix. From Eq. (11), \( A^T B^T M = \text{ABM} = 0 \) because A and B are symmetric matrices. Consequently, each row vector of the AB matrix is in the null space of M (i.e., each row vector of AB is a linear combination of \( e_j \) where \( e_j \) is the jth row of the E matrix). Therefore, the matrix AB can be expressed in terms of E and \( \rho^* \) as follows:

\[
\begin{align*}
AB &= \begin{bmatrix}
\alpha_1 e_1 + \cdots + \alpha_{m-1} e_{m-1} + \alpha_m \rho^* \\
\alpha_2 e_1 + \cdots + \alpha_{m-1} e_{m-1} + \alpha_m \rho^* \\
\vdots \\
\alpha_m e_1 + \cdots + \alpha_m e_{m-1} + \alpha_m \rho^*
\end{bmatrix} \\
&= \begin{bmatrix}
\alpha_1^{(m \times n)} \\
\alpha_2^{(n \times m-n)} \\
\alpha_m^{(n \times m-n)}
\end{bmatrix}
\begin{bmatrix}
E \\
\rho^*
\end{bmatrix}
\end{align*}
\]

(14)

where the \( \alpha \)'s are proportionality factors; \( m = m+1 \); and L=E matrix with one additional row containing \( \rho^* \).

Multiplying by \( B^{-1} \) on both sides yields

\[ A = \begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_m
\end{bmatrix}
\begin{bmatrix}
B^{-1} \\
G_1^{(m \times n)} \\
G_2^{(m \times n)}
\end{bmatrix}
\begin{bmatrix}
G_1 \\
G_2
\end{bmatrix}
\]

(15)

where G=LB^{-1}.

Because A is symmetric, one may write

\[ (\alpha_1 G_1, \alpha_2 G_2)^T G_1 = \alpha_1 G_1 \]

(16)

\[ -\alpha_2 = G_2^T \alpha_1 \]

(17)

Combining Eqs. (16) and (17) gives

\[ A = \begin{bmatrix}
\alpha_1 \\
G_2^T \alpha_1
\end{bmatrix}
\begin{bmatrix}
G_1 \\
G_2
\end{bmatrix}
\]

(18)

Because G is known, \( \alpha_1 \) is the only unknown in Eq. (18). Eq. (16) finds

\[ \Delta \rho^* = G_1^T \alpha_1 G_1 \rho^* = B^{-1} \rho^* \iff \rho^* = B^{-1} G_1^T \alpha_1 G_1 \rho^* \]

(19)

\[ \rho^* = G_1 G_2 \begin{bmatrix}
\rho_1^{(m \times 1)} \\
\rho_2^{(n \times m-n)}
\end{bmatrix} = G_1 \rho_1^* + G_2 \rho_2^* \]

(20)

Let C be \( B^{-1} G_1^T \), then

\[ C = B^{-1} G_1^T \]

(21)

Combining Eqs. (19) and (21) gives

\[ \begin{bmatrix}
\rho_1^* \\
\rho_2^*
\end{bmatrix} = \begin{bmatrix}
C_1 \\
C_2
\end{bmatrix} \begin{bmatrix}
\rho_1^{(m \times 1)} \\
\rho_2^{(n \times m-n)}
\end{bmatrix} = C_1 \rho_1^* + C_2 \rho_2^* \]

(22)

Price can be found explicitly as follows

\[ \rho^* = \begin{bmatrix}
\rho_1^* \\
\rho_2^*
\end{bmatrix} = \begin{bmatrix}
C_1 \\
C_2
\end{bmatrix} \begin{bmatrix}
\rho_1^{(m \times 1)} \\
\rho_2^{(n \times m-n)}
\end{bmatrix} = \begin{bmatrix}
C_1 \\
C_2
\end{bmatrix} \begin{bmatrix}
\rho_1^{(m \times 1)} \\
\rho_2^{(n \times m-n)}
\end{bmatrix} \begin{bmatrix}
\rho_1^{(m \times 1)} \\
\rho_2^{(n \times m-n)}
\end{bmatrix} = \rho_1^* + \rho_2^* \]

(23)

By substituting Eq. (21) into Eq. (23), \( \rho^* \) is

\[ \rho^* = (B^{-1} G_1^T)(B_1 G_1 G_1^T)^{-1} \rho_1 = B^{-1} G_1^T (B_1 G_1 G_1^T)^{-1} \rho_1 = B^{-1} G_1^T L^{-1} L^T (B_1 G_1 G_1^T)^{-1} \rho_1 \]

(24)

Eqs. (22) and (24) can be combined to yield

\[ C_1 \alpha_1 G_1 \rho^* = C_1 \alpha_1 [B (B_1 G_1 G_1^T)^{-1} \rho_1] \]

(25)

Eq. (25) is valid for all \( \rho_1^* \), which means

\[ \alpha_1 = C_1^{-1} [B (B_1 G_1 G_1^T)^{-1}] = C_1^{-1} G_1^T (B_1 G_1 G_1^T)^{-1} \]

(26)

The A and M matrices can be found by combining Eqs. (18) and (26)

\[ A = G_1^T G_1 \alpha_1 \]

(27)

\[ M = B^{-1} \frac{L}{\mu} \]

\[ L = \begin{bmatrix}
1 \\
\vdots \\
\mu
\end{bmatrix}
\]

(28)

For the rearranged vectors, a new M matrix can be defined as

\[ \Delta g_{\text{new}} = \mu \Delta g \]

(29)

\[ \Delta \rho_{\text{new}} = \mu \Delta \rho \]

(30)

Because the dispatched quantities of the "must-run" units are not affected by \( \rho^* \), the new M matrix is then

\[ M_{\text{new}} = \begin{bmatrix}
\mu \alpha_1 \chi \times \mu \alpha_2 \chi \times \mu \end{bmatrix}
\]

(31)

This rearrangement essentially separates the system into two subsystems where a change in one does not affect the other. Consequently, \( M_{\text{reduced}} \) can be estimated by applying Eq. (28) with
reduced $E$, $B$, and $p^*$ that exclude the $y$-must-run generators. Finally, the full $M$ matrix can be reconstructed from the matrix $M_{new}$ by computing

$$M = P^T M_{new} P$$  \hspace{1cm} (32)

Due to the separation, the space spanned by $M$ is divided into two subspaces of dimensions $(n_g - y)$ and $y$ so that the two subspaces are independent. The eigenvectors of $M$ are therefore divided into two groups that span one of the two subspaces. Eigenvectors spanning the $(n_g - y)$ dimensional subspace contain zero elements for $y$-injections, and those spanning the $y$-dimensional subspace do so for $(n_g - y)$ injections. It is also worthwhile to note that each of the $y$-injections are separated from each other. Therefore, eigenvectors spanning the $y$-dimensional subspace are unit vectors parallel to the price of the $y$-injections. The eigenvalues related to the separation are zero because the eigenvectors span the null space of $M$.

To estimate the $M$ matrix, $p^*$ needs to be evaluated. If demand is located at a single bus, then $p^*$ is the nodal price at the bus. However, in general, demand is distributed throughout the network. Consequently, it is difficult to define an aggregate shadow price of the power balance equation. By definition, $p^*$ is the cost to deliver one more unit of real power to an aggregated load bus. Because there are different probabilities for each demand bus to require one more unit, $p^*$ can be evaluated by using demand profile and a shadow price for each demand as

$$p^* = \sum \left( \frac{D_i}{D_{total}} \right) p_i^*$$  \hspace{1cm} (33)

where $D_i$ stands for the real power demand at the $i$th bus.

Revenue sensitivity matrix $N$, according to change in $p^*$, can be calculated by

$$\Delta p_i = \Delta (p_i^* g_i^* g_i^*) = g_i^* \Delta p_i^* = p_i^* \left( \sum_j M_{ij} \Delta p_j^* \right) + \Delta p_i^*$$

$$= \left[ \text{diag}(p_i^*) M \Delta p^* \right]_i + \left[ \text{diag}(g_i^*) \Delta p^* \right]_i$$

$$\Delta p^* = \left[ \text{diag}(p_i^*) M + \text{diag}(g_i^*) \right] \Delta p^* = N \Delta p^*$$  \hspace{1cm} (34)

If an agent owns multiple generators, Eq. (35) can indicate whether it has market power as a group.

**Case Studies**

To test the accuracy of the estimation method shown in Eq. (28), the modified IEEE 30-bus system shown in Fig. 1 is used. For the system, true and estimated $M$ matrices for various offers are calculated. Table 1 shows a comparison between them when no line is congested, and consequently no market power exists. In the simulation, the dispatched quantity and prices are [25.00, 45.00, 60.00, 2.22, 6.66, and 12.00 MW] and [83.31, 82.81, $21.00, $35.00, 276.68, and 168.51/MWh], respectively. As expected, the error between the true and estimated $M$ matrices becomes larger than in the previous case. However, the estimated $M$ matrix is still reasonably accurate as an indicator of who has market power. In this case, no individual generator has market power (see the "Market Power Monitor" section for a detailed discussion).

While reactive power flow over a line is not considered in this study, in general, reactive power generation and flow may also be changed when price is disrupted. For some cases, the change is not negligible. However, Eq. (28) still approximates the true $M$ matrix well if the change is approximately proportionate to that of real power because Eq. (13) still holds. Note that $\Delta w = \Delta w_{reactive} = \Delta w_{reactive} + \epsilon \Delta e = (\epsilon + 1) \Delta e = 0$ where $\Delta w_{reactive}$ and $\epsilon$ stand for the change in reactive injection and constant, respectively.

Table 2 shows the results of the case when the change in reactive power flow over the congested lines is substantial and is not proportionate to that in real power flow. In this case, the flow limit of the top tie-line is reduced to 10 MVA. The dispatched quantity and prices are [37.92, 34.92, 30.13, 34.90, 14.94, and 14.61 MW] and [84.48, 84.73, 48.49, 48.59, 72.03, and 70.00/MWh], respectively. Even though the error is substantial, the main purpose of estimating $M$ matrix is to estimate market power. Therefore, it is useful to check to see whether the estimated $M$ matrix indicates potential market power. For this purpose, the revenue sensitivity matrices are evaluated and shown in Table 4 for the same case by using Eq. (35). In this case, there exists a positive change in the price of Generators G5 and G6, which will cause their revenue increase. Consequently, they are able to raise prices. In other words, elements of the true $N$ matrix for G5 and G6 have a positive subset sum. The situation indicates a "win-win" situation (Lesieutre et al. 2003). The $N$ matrix derived from the estimated $M$ also indicates the same re-
results. Figs. 3 and 4 illustrate the same curves as shown in Fig. 2 for the cases discussed in Tables 2 and 3, respectively.

Market Power Monitor

According to the Federal Energy Regulatory Commission proposal (U.S. Federal Energy Regulatory Commission 2002), market power is defined by the ability to increase price above the competitive level. However, the “competitive” level is difficult to evaluate when generation production costs are not readily available.

As mentioned in the “Introduction” section, market power exists in cases of nonsubstitutability. In this section, nonsubstitutability, along with the definition of market power (i.e., an ability to raise price profitably), is addressed using an \( M \) matrix to detect such potential.

\[
\Delta g^* = M \Delta p^* \\
\text{where } \Delta p^* > 0 \quad (36)
\]

In Eq. (36), all the elements in \( \Delta p \) are positive. The use of the \( M \) matrix is for assessing market power, which is directly related to entries in the \( N \) matrix defined in Eq. (35). For example, to examine the existence of market power by the pair of Generators (G2 and G6) for the case where \( N \) matrix is shown in Table 4, the sub-\( N \) matrix of G2 and G6, which is \([-3683, -216.9; -311.6, -169.2]\), needs to be considered. It is possible to construct regions for the revenue of G2 or G6 to increase with respect to the change in their \( p^* \) as follows:

\[
-3,683\Delta p_{G2} - 216.9\Delta p_{G6} > 0
\]

\[
-311.6\Delta p_{G2} - 169.2\Delta p_{G6} > 0 \quad (37)
\]

If they decrease their \( p^* \), their revenues increase simultaneously. For example, suppose their change in \( p^* \) is \([-1; -1]\), then their revenue increase can be evaluated as \([-3,683, -216.9; -311.6, -169.2]\) \([-1; -1]=[-33.3, 999.9; 480.8]\). No positive change in locational marginal price (LMP) yields revenue increases of both generators simultaneously; therefore, they do not have market power. The dispatch change can also be calculated by using sub-\( M \) matrix from Table 3 \([-76.29, -4.45; -4.45, -2.63]\) \([-1; -1]=[-81.74, 7.08]\), which is a large perturbation. Large perturbations in dispatch due to a small price perturbation imply they are substitutable units.

Fig. 2. Residual demand curves of: (a) Generator 3; (b) Generator 5 as a function of the price of Generator 5 in the case used to calculate Table 1.
Table 2. Comparison between True and Estimated $M$ Matrices for the Case Where Several Lines Are Congested in the Unit of [MW/($/\text{MWh})$]; 2-Norm Error between Two Matrices is 0.115

<table>
<thead>
<tr>
<th>True Nest</th>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$G_3$</th>
<th>$G_4$</th>
<th>$G_5$</th>
<th>$G_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>-42.11</td>
<td>37.57</td>
<td>1.11</td>
<td>1.89</td>
<td>0.46</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>-45.06</td>
<td>38.38</td>
<td>-2.96</td>
<td>7.59</td>
<td>1.12</td>
<td>0.37</td>
</tr>
<tr>
<td>$G_2$</td>
<td>37.57</td>
<td>-53.73</td>
<td>6.36</td>
<td>3.80</td>
<td>0.51</td>
<td>5.41</td>
</tr>
<tr>
<td></td>
<td>38.38</td>
<td>-48.96</td>
<td>5.54</td>
<td>1.56</td>
<td>0.75</td>
<td>2.84</td>
</tr>
<tr>
<td>$G_3$</td>
<td>1.11</td>
<td>6.36</td>
<td>-17.07</td>
<td>13.20</td>
<td>-1.18</td>
<td>-2.35</td>
</tr>
<tr>
<td></td>
<td>-2.96</td>
<td>5.54</td>
<td>-11.63</td>
<td>10.56</td>
<td>-0.75</td>
<td>-0.77</td>
</tr>
<tr>
<td>$G_4$</td>
<td>1.89</td>
<td>3.80</td>
<td>13.20</td>
<td>-18.51</td>
<td>-0.23</td>
<td>-0.22</td>
</tr>
<tr>
<td></td>
<td>7.59</td>
<td>1.56</td>
<td>10.56</td>
<td>-18.14</td>
<td>-1.62</td>
<td>0.59</td>
</tr>
<tr>
<td>$G_5$</td>
<td>0.46</td>
<td>0.51</td>
<td>-1.18</td>
<td>-0.23</td>
<td>-1.09</td>
<td>1.51</td>
</tr>
<tr>
<td></td>
<td>1.12</td>
<td>0.75</td>
<td>-0.75</td>
<td>-1.62</td>
<td>-2.17</td>
<td>3.07</td>
</tr>
<tr>
<td>$G_6$</td>
<td>1.07</td>
<td>5.41</td>
<td>-2.35</td>
<td>-0.22</td>
<td>1.51</td>
<td>-5.32</td>
</tr>
<tr>
<td></td>
<td>0.37</td>
<td>2.84</td>
<td>-0.77</td>
<td>0.59</td>
<td>3.07</td>
<td>-6.65</td>
</tr>
</tbody>
</table>

However, suppose one considers the pair of $G_5$ and $G_6$. The sub-$N$ matrix is $[-172.3, 188.1; 182.8, -169.2]$. Similar to Eq. (37), one can formulate an equation relating revenue change with $\rho^*$ change

$$-172.3\Delta\rho_{G_5}^* + 188.1\Delta\rho_{G_6}^* > 0$$

Then the price change $[1; 1]$, which is a $1 \text{ MWh}$ increase in their price, increases their revenue (i.e., they are able to raise price profitably). Nonsubstitutability for the example above can also be checked by using sub-$M$ matrix; $[-2.60, 2.61; 2.61, -2.63]$

Table 3. Comparison between True and Estimated $M$ Matrices for the Case Where Several Lines Are Congested and Some Generators Have Market Power in the Unit of [MW/($/\text{MWh})$]; 2-Norm Error between Two Matrices Is 0.048

<table>
<thead>
<tr>
<th>True Nest</th>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$G_3$</th>
<th>$G_4$</th>
<th>$G_5$</th>
<th>$G_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>-68.71</td>
<td>66.46</td>
<td>0.28</td>
<td>1.57</td>
<td>0.79</td>
<td>-0.77</td>
</tr>
<tr>
<td></td>
<td>-64.75</td>
<td>66.56</td>
<td>-2.12</td>
<td>-0.00</td>
<td>-0.69</td>
<td>0.69</td>
</tr>
<tr>
<td>$G_2$</td>
<td>66.46</td>
<td>-76.29</td>
<td>4.92</td>
<td>5.14</td>
<td>4.43</td>
<td>-4.45</td>
</tr>
<tr>
<td></td>
<td>66.56</td>
<td>-78.40</td>
<td>6.28</td>
<td>5.68</td>
<td>6.36</td>
<td>-6.36</td>
</tr>
<tr>
<td>$G_3$</td>
<td>0.28</td>
<td>4.92</td>
<td>-12.34</td>
<td>7.24</td>
<td>-3.12</td>
<td>3.11</td>
</tr>
<tr>
<td></td>
<td>-2.12</td>
<td>6.28</td>
<td>-8.80</td>
<td>4.73</td>
<td>-3.31</td>
<td>3.31</td>
</tr>
<tr>
<td>$G_4$</td>
<td>1.57</td>
<td>5.14</td>
<td>7.24</td>
<td>-13.88</td>
<td>-2.03</td>
<td>2.04</td>
</tr>
<tr>
<td></td>
<td>-0.00</td>
<td>5.68</td>
<td>4.73</td>
<td>-10.32</td>
<td>-2.25</td>
<td>2.25</td>
</tr>
<tr>
<td>$G_5$</td>
<td>0.79</td>
<td>4.43</td>
<td>-3.12</td>
<td>-2.03</td>
<td>-2.60</td>
<td>2.61</td>
</tr>
<tr>
<td></td>
<td>-0.69</td>
<td>6.36</td>
<td>-3.31</td>
<td>-2.25</td>
<td>-3.33</td>
<td>3.33</td>
</tr>
<tr>
<td>$G_6$</td>
<td>-0.77</td>
<td>-4.45</td>
<td>3.11</td>
<td>2.04</td>
<td>2.61</td>
<td>-2.63</td>
</tr>
<tr>
<td></td>
<td>0.69</td>
<td>-6.36</td>
<td>3.31</td>
<td>2.25</td>
<td>3.33</td>
<td>-3.33</td>
</tr>
</tbody>
</table>

Table 4. Comparison between True and Estimated $N$ Matrices for the Case Where Several Lines Are Congested and Some Generators Have Market Power in the Unit of [$/\text{h}/($/\text{MWh})$]

<table>
<thead>
<tr>
<th>True Nest</th>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$G_3$</th>
<th>$G_4$</th>
<th>$G_5$</th>
<th>$G_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>-3,293</td>
<td>3,222</td>
<td>13.5</td>
<td>76.3</td>
<td>38.2</td>
<td>-37.5</td>
</tr>
<tr>
<td></td>
<td>-3,101</td>
<td>3,227</td>
<td>-102.7</td>
<td>-0.1</td>
<td>-33.5</td>
<td>33.5</td>
</tr>
<tr>
<td>$G_2$</td>
<td>3,239</td>
<td>-3,683</td>
<td>239.6</td>
<td>250.4</td>
<td>215.9</td>
<td>-216.9</td>
</tr>
<tr>
<td></td>
<td>3,243</td>
<td>-3,785</td>
<td>306.1</td>
<td>277.0</td>
<td>309.8</td>
<td>-309.8</td>
</tr>
<tr>
<td>$G_3$</td>
<td>13.5</td>
<td>238.5</td>
<td>-568.1</td>
<td>351.0</td>
<td>-151.2</td>
<td>159.0</td>
</tr>
<tr>
<td></td>
<td>-102.7</td>
<td>304.6</td>
<td>-396.4</td>
<td>292.3</td>
<td>-160.3</td>
<td>160.3</td>
</tr>
<tr>
<td>$G_4$</td>
<td>78.5</td>
<td>249.7</td>
<td>351.7</td>
<td>639.7</td>
<td>-98.5</td>
<td>99.3</td>
</tr>
<tr>
<td></td>
<td>-0.2</td>
<td>276.2</td>
<td>229.8</td>
<td>466.7</td>
<td>-109.1</td>
<td>109.1</td>
</tr>
<tr>
<td>$G_5$</td>
<td>58.6</td>
<td>319.1</td>
<td>-224.6</td>
<td>-146.1</td>
<td>-172.3</td>
<td>188.1</td>
</tr>
<tr>
<td></td>
<td>-49.8</td>
<td>457.9</td>
<td>-238.2</td>
<td>-161.7</td>
<td>-225.0</td>
<td>240.0</td>
</tr>
<tr>
<td>$G_6$</td>
<td>-54.2</td>
<td>-311.6</td>
<td>217.9</td>
<td>143.0</td>
<td>182.8</td>
<td>-169.2</td>
</tr>
<tr>
<td></td>
<td>-48.4</td>
<td>-44.5</td>
<td>231.5</td>
<td>157.2</td>
<td>233.2</td>
<td>-218.6</td>
</tr>
</tbody>
</table>
\[ \times [1; 1] = [-0.01; -0.02], \] which is approximately zero (i.e., G5 and G6 are nonsubstitutable units). The same analysis performed with the estimated \( N \) matrix yields an identical result with the true \( N \) matrix.

A similar analysis can be performed using \( N \) and \( M \) matrix for any subset of generators to assess market power. For other cases where \( M \) matrices are listed in Table 1 and 2, all possible combinations among six generators were considered. It was found that no subset of generators had market power and that all generators were substitutable.

In the economy, price elasticity is used to measure responsiveness in the quantity as a result of change in price of the commodity. Therefore, the price elasticity matrix \( PE \) can be derived using \( M \) as follows:

\[
PE = [\varepsilon_{ij}]
\]

where \( \varepsilon_{ij} = \frac{\frac{\partial q_i}{\partial p_j}}{\frac{\partial p_i}{\partial p_j}} = \left( \frac{1}{B_i} \right)[\frac{\partial g_i}{\partial p_j}](p) = \left( \frac{1}{B_i} \right)M_{ij}(p)
\]

\[
= \left( \frac{1}{B_i} \right)e_i^TM_{ij}(p) = \left( \frac{1}{B_i} \right)e_i^TM(p_{i,j})\]

\( \rightarrow PE = [\text{diag}(g)]^{-1}M[\text{diag}(\lambda)] \) \quad (39)

Muirillo-Sanchez et al. (2001) showed that revenue and profit for generator \( i \) in a group will increase by the raising price if

\[
\sum_{j \neq i} e_{ij} > -1\) \quad (40)

Tables 5–7 show the price elasticity matrix using the true and estimated \( M \) matrices. Both true and estimated \( PE \) matrices clearly show that Eq. (40) is satisfied for the group comprised of G5 and G6.

Conclusions

In this paper, a method for detecting market power based on a dispatch-to-price sensitivity matrix is proposed, showing that computing an estimate of the sensitivities requires only publically available data. Through case studies, it can be conclude that the method works reasonably well. Some cases have a larger error than others, but it still does a good job of identifying potential
Table 5. Comparison between True and Estimated PE Matrices for the Case Where No Line Is Congested; Note That PE Matrix Is Dimensionless, and 2-Norm Error between Two Matrices Is 0.046

<table>
<thead>
<tr>
<th>PEtrue</th>
<th>PEest</th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
<th>G5</th>
<th>G6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G1</td>
<td></td>
<td>-141.8</td>
<td>113.2</td>
<td>7.26</td>
<td>4.77</td>
<td>-0.06</td>
<td>16.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-140.64</td>
<td>111.8</td>
<td>7.57</td>
<td>6.31</td>
<td>-3.11</td>
<td>18.09</td>
</tr>
<tr>
<td>G2</td>
<td></td>
<td>62.83</td>
<td>-103.0</td>
<td>12.63</td>
<td>8.70</td>
<td>-0.10</td>
<td>18.95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>62.04</td>
<td>-104.9</td>
<td>11.71</td>
<td>8.35</td>
<td>3.85</td>
<td>18.77</td>
</tr>
<tr>
<td>G3</td>
<td></td>
<td>3.07</td>
<td>9.62</td>
<td>-17.00</td>
<td>1.38</td>
<td>2.70</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.20</td>
<td>8.92</td>
<td>-18.97</td>
<td>2.49</td>
<td>3.49</td>
<td>0.86</td>
</tr>
<tr>
<td>G4</td>
<td></td>
<td>2.02</td>
<td>6.51</td>
<td>1.38</td>
<td>-12.58</td>
<td>1.52</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.66</td>
<td>6.50</td>
<td>2.49</td>
<td>-13.72</td>
<td>1.06</td>
<td>1.01</td>
</tr>
<tr>
<td>G5</td>
<td></td>
<td>-0.06</td>
<td>0.19</td>
<td>6.71</td>
<td>3.79</td>
<td>-25.09</td>
<td>14.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-3.26</td>
<td>7.25</td>
<td>8.68</td>
<td>2.64</td>
<td>-28.71</td>
<td>13.41</td>
</tr>
<tr>
<td>G6</td>
<td></td>
<td>33.12</td>
<td>68.25</td>
<td>1.13</td>
<td>4.98</td>
<td>28.27</td>
<td>-135.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>36.17</td>
<td>67.59</td>
<td>4.08</td>
<td>4.79</td>
<td>25.53</td>
<td>-138.2</td>
</tr>
</tbody>
</table>

market power. The mathematical relationship between the true M matrix and the estimated one—that is, the error between them—is still a topic of research.

Appendix. M for a Lossless System

The M matrix in this study is the sensitivity of bus injection with respect to LMP. Consequently, bus injection needs to be distin-

Table 6. Comparison between True and Estimated PE Matrices for the Case Where Several Lines Are Congested; Note That PE Matrix Is Dimensionless and 2-Norm Error between Two Matrices Is 0.136

<table>
<thead>
<tr>
<th>PEtrue</th>
<th>PEest</th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
<th>G5</th>
<th>G6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G1</td>
<td></td>
<td>-140.3</td>
<td>124.5</td>
<td>0.94</td>
<td>2.65</td>
<td>5.04</td>
<td>7.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-150.1</td>
<td>127.1</td>
<td>-2.49</td>
<td>10.63</td>
<td>12.37</td>
<td>2.50</td>
</tr>
<tr>
<td>G2</td>
<td></td>
<td>69.56</td>
<td>-98.87</td>
<td>2.97</td>
<td>2.96</td>
<td>3.14</td>
<td>20.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>71.06</td>
<td>-90.09</td>
<td>2.58</td>
<td>1.21</td>
<td>4.61</td>
<td>10.62</td>
</tr>
<tr>
<td>G3</td>
<td></td>
<td>1.70</td>
<td>9.64</td>
<td>-6.56</td>
<td>8.45</td>
<td>-5.99</td>
<td>-7.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-4.52</td>
<td>8.39</td>
<td>-4.47</td>
<td>6.75</td>
<td>-3.80</td>
<td>-2.36</td>
</tr>
<tr>
<td>G4</td>
<td></td>
<td>3.12</td>
<td>6.23</td>
<td>5.49</td>
<td>-12.83</td>
<td>-1.27</td>
<td>-0.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12.52</td>
<td>2.55</td>
<td>4.39</td>
<td>-12.58</td>
<td>-8.87</td>
<td>1.98</td>
</tr>
<tr>
<td>G5</td>
<td></td>
<td>1.52</td>
<td>1.69</td>
<td>-0.99</td>
<td>-0.33</td>
<td>-12.05</td>
<td>10.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.72</td>
<td>2.48</td>
<td>-0.63</td>
<td>-2.27</td>
<td>-24.02</td>
<td>20.71</td>
</tr>
<tr>
<td>G6</td>
<td></td>
<td>3.58</td>
<td>17.90</td>
<td>-1.97</td>
<td>0.31</td>
<td>16.69</td>
<td>-35.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.24</td>
<td>9.39</td>
<td>-0.64</td>
<td>0.83</td>
<td>34.00</td>
<td>-44.81</td>
</tr>
</tbody>
</table>

Table 7. Comparison between True and Estimated PE Matrices for the Case Where Several Lines Are Congested and Some Generators Have Market Power; Note That PE Matrix Is Dimensionless, and 2-Norm Error between Two Matrices Is 0.079

<table>
<thead>
<tr>
<th>PEtrue</th>
<th>PEest</th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
<th>G5</th>
<th>G6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G1</td>
<td></td>
<td>-87.85</td>
<td>85.41</td>
<td>0.36</td>
<td>2.02</td>
<td>1.50</td>
<td>-1.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-82.78</td>
<td>85.53</td>
<td>-2.71</td>
<td>-0.00</td>
<td>-1.31</td>
<td>1.28</td>
</tr>
<tr>
<td>G2</td>
<td></td>
<td>92.27</td>
<td>-106.5</td>
<td>6.83</td>
<td>7.15</td>
<td>9.14</td>
<td>-8.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>92.40</td>
<td>-109.4</td>
<td>8.72</td>
<td>7.91</td>
<td>13.11</td>
<td>-12.74</td>
</tr>
<tr>
<td>G3</td>
<td></td>
<td>0.45</td>
<td>7.95</td>
<td>-19.85</td>
<td>11.67</td>
<td>-7.45</td>
<td>7.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-3.41</td>
<td>10.16</td>
<td>-14.16</td>
<td>7.63</td>
<td>-7.91</td>
<td>7.58</td>
</tr>
<tr>
<td>G4</td>
<td></td>
<td>2.19</td>
<td>7.18</td>
<td>10.06</td>
<td>-19.33</td>
<td>-4.19</td>
<td>4.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.00</td>
<td>7.94</td>
<td>6.57</td>
<td>-14.37</td>
<td>-6.03</td>
<td>4.50</td>
</tr>
<tr>
<td>G5</td>
<td></td>
<td>2.56</td>
<td>14.45</td>
<td>-10.12</td>
<td>-6.60</td>
<td>-12.53</td>
<td>12.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2.24</td>
<td>20.73</td>
<td>-10.73</td>
<td>-7.30</td>
<td>-16.06</td>
<td>15.61</td>
</tr>
<tr>
<td>G6</td>
<td></td>
<td>-2.57</td>
<td>-14.85</td>
<td>10.33</td>
<td>6.79</td>
<td>12.87</td>
<td>-12.58</td>
</tr>
</tbody>
</table>
Notation

The following symbols are used in this paper:

- \( B \) = loss matrix determined at the optimal dispatch;
- \( \text{Cost} \) = objective function for the first (ac) OPF;
- \( \mathbf{e} \) = vector of transmission line capacities (MW);
- \( \mathbf{d} \) = real power demand vector;
- \( \mathbf{E} \) = submatrix of \( \mathbf{H} \) associated with binding line constraints;
- \( \mathbf{F} \) = vector of equality constraints;
- \( \mathbf{G} \) = vector of inequality constraints;
- \( \mathbf{g} \) = net real power injection vector at Generation Bus I (MW);
- \( \mathbf{g}^* \) = vector of optimal generator real power injections (MW);
- \( \mathbf{H} \) = PTDF matrix;
- \( \mathbf{I} \) = identity matrix;
- \( k \) = number of transmission lines;
- \( n_g \) = dimension of vector \( g \);
- \( \mathbf{P} \) = permutation matrix;
- \( \mathbf{P}_{\text{ tot}} \) = total real power demand (MW);
- \( \mathbf{P}_{\text{loss}} \) = real power loss vector (MW);
- \( \text{PE} \) = price elasticity matrix;
- \( q_j \) = generator \( j \)'s offer quantity (MW);
- \( r_j \) = generator \( j \)'s revenue;
- \( x \) = ac OPF control variables such as real and reactive power, voltage angle, and magnitude;
- \( z \) = vector of generator real power injections (MW);
- \( \eta_{\text{K}} \) = equality constraints of Kuhn-Tucker multipliers;
- \( \eta_{\text{I}} \) = vector of inequality constraint Kuhn-Tucker multipliers;
- \( \lambda_j \) = generator \( j \)'s offer price ($/MWh);
- \( \mu \) = total power balance constraint shadow price ($/MWh);
- \( \rho^* \) = locational marginal price vector; and
- \( \sigma \) = vector of congested lines shadow prices ($/MWh).

Fig. 5. Residual demand curves of: (a) Generator 3; (b) Generator 5 as a function of the price of Generator 5 in the case used to calculate Table 3 by using a lossless DC power flow model.

5 shows the residual demand curves of the case illustrated in Fig. 2 in a lossless direct current network. Clearly, the residual demand curves are a pure step function; therefore, the slopes (the elements of \( M \)) are zeros or negative infinity.

For a radial network, a congested transmission line separates a system into two subsystems, of which \( \rho^* \) can be different. Price separation also occurs for a meshed network. In other words, each subsystem has its own price setter. A system can be separated into multiple pieces according to congestion and merge buses if they belong to an identical subsystem. Suppose a lossless system contains \( m \) congested lines, and consequently \( m \)-subsystems. Then, the dispatch—including price and quantities sold in one subsystem—does not affect the dispatch in another subsystem.

Each subsystem has one merged generator monopoly; therefore, the dispatched quantity of the generator is not affected by its own or other generators’ prices. Likewise, other generators’ dispatched quantities are not affected by its LMP. Consequently, corresponding row and column vectors to the generator are zero. Therefore, \( M \) matrix has all zero entries with dimension of \( m \)-by-\( m \). Note that it is not possible to convert \( M \) matrix to full dimension of \( n_g \)-by-\( n_g \) matrix due to nonseparable buses in a lossless system.

References


