Co-optimization of transmission maintenance scheduling and production cost minimization

Abstract

This study focuses on the co-ordination of transmission outage scheduling with day-ahead energy markets. It is developed to schedule transmission line maintenance requests optimally and centrally at the system operator level. The preliminary results promise up to 4% production cost savings by shifting the maintenance duration to an optimal time frame when the loss of this transmission line reduces the total operating cost of the system. The model co-optimizes generation unit commitment and transmission line outage coordination with N-1 reliability. The model is flexible to be converted to a security constrained unit commitment or to an optimal topology control problem, and its effectiveness is tested on a modified IEEE 30-bus system. Financial benefits of the model are discussed in detail, and compared with those from Business-As-Usual model as adopted.

Keywords: Outage Scheduling, Transmission topology control, Unit commitment, N-1 reliability.

1. Introduction

Unit commitment commits generators to meet the power demand while minimizing the production cost of system [1-2]. It is generally formulated in the form of Mixed Integer Linear Programming (MILP). Several solution techniques are discussed in the literature [3-6], including Lagrangian relaxation [7, 8], Branch & Bound [9, 10, 11], and their comparisons [12, 13, 14]. Extended surveys of unit commitment studies are also presented in Refs [15, 16].

Transmission switching program, or Optimal Topology Control (OTC), selects and keeps transmission lines in service to minimize production cost [17, 18]. Similar to a UC program, OTC is also formulated in the form of MILP. Although its theoretical background is studied and published by several groups [19-28], there is no reported use of OTC in business. However, possible applications of OTC in deregulated markets are discussed in Ref. [29]. The main reason is the challenges that haven’t been fully addressed.

This study, however, adopts the financial benefits of optimal topology control and applies them on an already present business model in outage coordination. By doing that, the challenges that are discussed on integrating OTC to the power market are no longer relevant to the proposed model. In a simplified way, the model schedules short-term maintenance of transmission lines in a centralized way, such that the execution of maintenance at the optimal time frame reduces the production cost of the system.

The main difference of the proposed model and the current scheduling programs is the transmission line scheduling studies [30] are generally presented from a transmission operator’s
The objective function either be the maximization of profit or the minimization of maintenance cost [31, 32], then the problems are solved by a self-scheduling method [33, 34] so no central co-ordination has been considered in the literature. This study; however, fills this gap by introducing a single objective optimization problem that co-ordinate the benefits of unit commitment, maintenance scheduling, and transmission switching in a production cost simulation framework.

Another contribution of this study is a new mathematical formulation for N-1 reliability constraint without a separate binary variable for contingencies. The new formulation reduces the cardinality of the problem significantly since the first development by Hedman [35].

2. Flexibility & Convertibility of the model

The proposed model is developed to co-optimize generation unit commitment with transmission switching from an outage coordination perspective. Cost savings are achieved by shifting the maintenance duration to an optimal time frame when the loss of this transmission line reduces the total operating cost of the system. The model captures the impact of a transmission line status on the production cost, so the optimal solution will always yield a better solution than the Business-as-usual model.

The model includes all constraints of an SCUC problem. In order to convert the model to an SCUC, as given in (P2), we drop the maintenance scheduling constraints given in (5a-5d) from the constraint set and set no pending maintenance requests (Ψ = ∅). Moreover, the proposed model can also be modified to be the exact problem of co-optimization of generation unit commitment and transmission switching with N-1 reliability [35]. Dropping the maintenance scheduling constraints given in (5a-5d) from the constraint set and selecting all transmission lines for maintenance (Ψ = Ω) converts the model to an OTC – SCUC, as presented in (P3).

The smallest feasible region of the proposed model is equivalent to that of SCUC, and the largest is equal to that of OTC-SCUC. Illustrative feasible regions are shown in Fig. 1. The feasible region of the model is always in between the feasible regions of these two problems. The size, however, depends solely on the number of branches waiting for maintenance.

The model can also be modified to show the tradeoff between total operating costs and the approval of one more request from the maintenance waiting list. This capability is unique and may be used to identify the Pareto optimal solutions from a multi-objective optimization perspective. The modified problem is given in (P4) by adding a linear equality constraint as shown in (7). The summation of request status variable \( a \) can be bounded by the total number of the approval parameter, \( N_{\text{approve}} \). The scalar value of \( N_{\text{approve}} \) may be selected from the set of \( \{0, 1, ..., n(\Psi)\} \) where \( n(\Psi) \) denotes the number of elements in Set \( \Psi \). The difference between two objective values may give an opportunity to the system operator to analyze the impact of maintenance on the total operating cost.
3. Problem Formulation

This section covers the mathematical representation of the proposed model so it has dense mathematics. Definitions of the variables are given in the Appendix. Generally, variables are denoted with three subscripts. They denote index, time, and contingency state parameters respectively. Other variables are denoted with two subscripts, which are index and time parameters. Finally, if the value of a parameter doesn’t change with respect to time, it has only one subscript for indexing.

The formulation includes linear equality and inequality constraints on power flow definitions, power balance equations, unit commitment, technical limits, and maintenance scheduling constraints. The objective is to minimize total production cost of the system. It is a summation of four different terms as shown in (1).

$$\begin{align*}
\min \quad & \sum_{i \in T} \sum_{g \in G} C_{s_i} P_{g,t} G_{t} B a s e^{MW} + \sum_{i \in T} \sum_{g \in G} N L_{g} u_{g,t} \\
& + \sum_{i \in T} \sum_{g \in G} S U_{g} s_{g,t} + \sum_{k \in K} P M_{k} \left( \sum_{i \in t} m_{k,t} - a_{k} \right) \\
\text{subject to} \quad & (2a - 2d) \\
& (3a - 3f) \\
& (4a - 4f) \\
& \Psi = \emptyset \\
& \Psi = \Omega \\
& \sum a_{k} = N_{\text{approve}} (7)
\end{align*}$$
The first term in (1) is production cost at steady state. The second term is no load cost. Together they represent the total cost of generation. The third term is startup cost (SU), which depends on the binary startup decision variable, s. The last term is partial maintenance (PM) cost. It treats all maintenance requests equally if they are scheduled in a block time frame; however, if scheduling the maintenance in two or more time windows leads to a better solution, then the model captures this tradeoff.

\begin{align*}
P_{kij,t,c} &\leq B_k \left( \theta_{i,t,c} - \theta_{j,t,c} \right) \left( 1 - z_{k,t} \right) M, \\
&\forall t, \forall k \in \Psi, \forall c \in X/k \quad (2a) \\
P_{kij,t,c} &\geq B_k \left( \theta_{i,t,c} - \theta_{j,t,c} \right) - \left( 1 - z_{k,t} \right) M, \\
&\forall t, \forall k \in \Psi, \forall c \in X/k \quad (2b) \\
-P_{kij,t,c}^\max &\leq P_{kij,t,c} \leq z_{k,t} P_{kij,t,c}^\max, \forall t, \forall k \in \Psi, \forall c \in X/k \quad (2c) \\
-P_{kij,t,c}^\max &\leq P_{kij,t,c} \leq P_{kij,t,c}^\max, \forall t, \forall k \in \Omega / \Psi, \forall c \in X/k \quad (2d)
\end{align*}

The constraints (2a), (2b), and (3a), (3b) define the amount of power flowing on a transmission line. Maximum flow limit of transmission lines is modeled in (2c) and (2d). The constraints (3e) and (3f) defines the power balance at each bus.

One of the contributions of this study is to reduce the size of the problem by eliminating a binary variable previously used in other studies [35]. Our observation is that the status of a transmission line; being ON or OFF, doesn’t change in any contingency states but for its own. Secondly, if the line is out due to a contingency then its transfer capability is zero. So it’s mathematically no longer a variable to be determined in the program but it’s already known. By introducing a parameter, we can avoid to introduce a separate variable as represented in the constraint (3b).

\begin{align*}
P_{kij,t,c} &\leq B_k \left( \theta_{i,t,c} - \theta_{j,t,c} \right), \forall t, \forall k \in \Omega / \Psi, \forall c \in X/k \quad (3a) \\
P_{kij,t,c} &\geq 0, \forall t, \forall k \in \Omega, \forall c = k \quad (3b) \\
\theta_{i,t,c} &\geq 0, \forall t, \forall i \in \text{ref}, \forall c \in X \quad (3c) \\
P_{g,t,c}^G &\geq 0, \forall t, \forall g \in \Gamma, \forall c = g \quad (3d) \\
\sum_g P_{g(t),t,c}^G &\leq P_{i,t}^D + \sum_{k \in (i,*)} P_{kij,t,c} - \sum_{k \in (*,i)} P_{kij,t,c} \\
&\forall t, \forall i \in \Phi^G, \forall c \in X \quad (3e) \\
0 &\leq P_{i,t}^D + \sum_{k \in (i,*)} P_{kij,t,c} - \sum_{k \in (*,i)} P_{kij,t,c} \\
&\forall t, \forall i \in \Phi / \Phi^G, \forall c \in X \quad (3f)
\end{align*}

Due to the adaptation of a lossless model in this study, the amount of power flowing from Bus \( i \) to Bus \( j \) is equal to the flow from Bus \( j \) to Bus \( i \). The set of \( k \in (i,*) \) in (3e) and (3f) includes branches having Bus \( i \) as their “From Bus” and the set of \( k \in (*,i) \) in (3e) and (3f) includes
branches having Bus \( i \) as their “To Bus”. Voltage angle for the reference bus set to zero to get a unique angle tensor at the optimal solution.

\[
P_{\text{g,min}}^{G} \leq P_{g,i,c}^{G} \leq P_{\text{g,max}}^{G} \quad \forall t, \forall g \in \Gamma, \forall c \in X / g \quad (4a)
\]

\[
-R_{g}^{DC} \leq P_{g,i,c}^{G} - P_{g,i,0}^{G} \leq R_{g}^{DC} \quad \forall t, \forall g \in \Gamma, \forall c \in X / g \quad (4b)
\]

\[
-R_{g}^{D} \leq P_{g,i,0}^{G} - P_{g,i,(t-1),0}^{G} \leq R_{g}^{D} \quad \forall t, \forall g \in \Gamma \quad (4c)
\]

\[
\sum_{t=1-p_{t}+1}^{t} s_{g,t} \leq u_{g,t} \quad \forall t, \forall g \in \Gamma \quad (4d)
\]

\[
\sum_{t=1-n_{t}+1}^{t} h_{g,t} \leq 1-u_{g,t} \quad \forall t, \forall g \in \Gamma \quad (4e)
\]

\[
s_{g,t} - h_{g,t} = u_{g,t} - u_{g,(t-1)} \quad \forall t, \forall g \in \Gamma \quad (4f)
\]

Several studies [6-14] use the constraints (4a-4f) to model the security constrained unit commitment. The constraint (4a) is for maximum and minimum generation capacity. (4b) is for a secure transition from a steady state to a contingency state. (4c) is for secure transition of generation dispatches between two consecutive time periods in a steady state. Two inequality constraints (4d, 4e) satisfy the minimum up and down time limits of each generator unit. The constraint (4f) is a valid constraint if and only if, either \( s \) or \( h \) appears in the objective function with a positive coefficient in front as the third term in (1) denoting the startup cost of generators.

\[
z_{k,(t-1)} - z_{k,t} \leq m_{k,t} \quad \forall t, \forall k \in \Psi \quad (5a)
\]

\[
\sum_{t=t-n_{t}+1}^{t} m_{k,t} \leq 1 - z_{k,t} \quad \forall t, \forall k \in \Psi \quad (5b)
\]

\[
\sum_{t=1}^{T} m_{k,t} \leq \ell_{k} a_{k} \quad \forall k \in \Psi \quad (5c)
\]

\[
T - \sum_{t=1}^{T} z_{k,t} = d_{k} a_{k} \quad \forall k \in \Psi \quad (5d)
\]

The last section of the model includes constraints on centralized maintenance scheduling. Worthwhile to note here that the set \( \Psi \) includes the transmission lines pending maintenance approval. All constraints on this section are defined solely for the set \( \Psi \). The constraint (5a) defines the binary variable \( m_{k,t} \) with respect to change in the status of Branch \( k \). It is valid if and only if \( m \) appears in the objective function with a positive coefficient.

The constraints (5b) and (5c) are included to add more flexibility in maintenance scheduling. A partial maintenance task may have a predetermined minimum time. This value may not be equal to the total duration required to complete the maintenance. Due to the fact that the model can split the total maintenance duration into partial time frames, the constraint (5b) ensures that Branch \( k \) kept open during the minimum time frame denoted by \( n_{k} \). However, a maintenance
task may not be possible to split. Some may be divided into one-hour intervals. To address this issue, the constraint \((5c)\) is included to have this flexibility defining any type of maintenance by changing the value of \(\ell_k\). For example, if \(\ell_k = 1\) then, the maintenance duration for Branch \(k\) cannot be divided into partial time frames. However, if \(\ell_k = d_k\), then the minimum partial maintenance window becomes a one-hour interval. The constraint \((5d)\) ensures that if the maintenance is approved, it has to be fully completed in the planning time horizon.

The final model can be created as a single objective optimization problem that minimizes \((1)\) subject to the constraints of Eq. \((2a-2d)\), \((3a-3f)\), \((4a-4f)\), and \((5a-5d)\).

5. Numerical example

A modified 30-bus system included in MATPOWER [36] is used in this study with minor modifications given in Table I. The dataset of the system is adopted from MATPOWER but additional parameters like startup cost, ramp rates, minimum up and down-time parameters are assumed to be given in Table II. Six transmission lines out of thirty-nine are assumed requesting maintenance (Table III). Hourly peak load in percent of total load (Table IV) is adopted from IEEE-Reliability Test System [37]. The 24-hour time period of interest is assumed to be a summer weekday with total peak load of 189.20 MW. Per-unit basis [38], \(Base^{MW}\), is set to 100 MW.

<table>
<thead>
<tr>
<th>Original Line ID</th>
<th>Modified Line ID</th>
<th>From Bus</th>
<th>To Bus</th>
<th>(\rho_k^{max})</th>
<th>Modification</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>15</td>
<td>4</td>
<td>12</td>
<td>0.39</td>
<td>Max. Flow limit</td>
</tr>
<tr>
<td>17</td>
<td>17</td>
<td>12</td>
<td>14</td>
<td>0.65</td>
<td>Max. Flow limit</td>
</tr>
<tr>
<td>18</td>
<td>18</td>
<td>12</td>
<td>15</td>
<td>0.65</td>
<td>Max. Flow limit</td>
</tr>
<tr>
<td>19</td>
<td>19</td>
<td>12</td>
<td>16</td>
<td>0.65</td>
<td>Max. Flow limit</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>14</td>
<td>15</td>
<td>0.32</td>
<td>Max. Flow limit</td>
</tr>
<tr>
<td>25</td>
<td>--</td>
<td>10</td>
<td>20</td>
<td>--</td>
<td>Line is removed</td>
</tr>
<tr>
<td>26</td>
<td>--</td>
<td>10</td>
<td>17</td>
<td>--</td>
<td>Line is removed</td>
</tr>
<tr>
<td>27-41</td>
<td>25-39</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>Modified Line ID</td>
</tr>
</tbody>
</table>

TABLE I
TRANSMISSION LINE MODIFICATIONS
TABLE II - GENERATOR PARAMETERS

<table>
<thead>
<tr>
<th>ID</th>
<th>Location, Bus ID</th>
<th>Unit Cost Coefficients</th>
<th>$p_{u_{\text{min}}}^{\text{g}}$</th>
<th>$p_{u_{\text{max}}}^{\text{g}}$</th>
<th>$w_{g}$</th>
<th>$p_{g}$</th>
<th>$R_{i}^{c} = R_{i}^{\alpha}$</th>
<th>$R_{i}^{cc} = R_{i}^{\alpha c}$</th>
<th>SU$_{g}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1</td>
<td>11.20</td>
<td>0.80</td>
<td>0</td>
<td>8</td>
<td>8</td>
<td>0.30</td>
<td>440</td>
<td></td>
</tr>
<tr>
<td>G2</td>
<td>2</td>
<td>10.80</td>
<td>0.80</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>0.31</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>G3</td>
<td>21</td>
<td>10.50</td>
<td>0.50</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>0.32</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>G4</td>
<td>27</td>
<td>10.20</td>
<td>0.55</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>0.29</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>G5</td>
<td>23</td>
<td>13.00</td>
<td>0.30</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0.35</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>G6</td>
<td>13</td>
<td>15.00</td>
<td>0.40</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0.40</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>

TABLE III - PARAMETERS OF LINES REQUESTING MAINTENANCE

<table>
<thead>
<tr>
<th>Priority</th>
<th>Line ID</th>
<th>From Bus</th>
<th>To Bus</th>
<th>$PM_{k}$</th>
<th>$d_{k}$</th>
<th>$\ell_{k}$</th>
<th>$r_{k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31</td>
<td>24</td>
<td>25</td>
<td>6</td>
<td>9</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>12</td>
<td>15</td>
<td>3</td>
<td>8</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>12</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>38</td>
<td>8</td>
<td>28</td>
<td>5</td>
<td>3</td>
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<td>9</td>
<td>6</td>
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<td>6</td>
<td>6</td>
<td>1</td>
</tr>
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<td>6</td>
<td>15</td>
<td>4</td>
<td>12</td>
<td>1</td>
<td>10</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

TABLE IV - HOURLY PEAK LOAD IN PERCENT OF TOTAL PEAK LOAD

<table>
<thead>
<tr>
<th>Hour 1</th>
<th>Hour 2</th>
<th>Hour 3</th>
<th>Hour 4</th>
<th>Hour 5</th>
<th>Hour 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>64%</td>
<td>60%</td>
<td>58%</td>
<td>56%</td>
<td>56%</td>
<td>58%</td>
</tr>
<tr>
<td>Hour 7</td>
<td>Hour 8</td>
<td>Hour 9</td>
<td>Hour 10</td>
<td>Hour 11</td>
<td>Hour 12</td>
</tr>
<tr>
<td>64%</td>
<td>76%</td>
<td>87%</td>
<td>95%</td>
<td>99%</td>
<td>100%</td>
</tr>
<tr>
<td>Hour 13</td>
<td>Hour 14</td>
<td>Hour 15</td>
<td>Hour 16</td>
<td>Hour 17</td>
<td>Hour 18</td>
</tr>
<tr>
<td>99%</td>
<td>100%</td>
<td>100%</td>
<td>97%</td>
<td>96%</td>
<td>96%</td>
</tr>
<tr>
<td>Hour 19</td>
<td>Hour 20</td>
<td>Hour 21</td>
<td>Hour 22</td>
<td>Hour 23</td>
<td>Hour 24</td>
</tr>
<tr>
<td>93%</td>
<td>92%</td>
<td>92%</td>
<td>93%</td>
<td>87%</td>
<td>72%</td>
</tr>
</tbody>
</table>

5.1. Results of proposed model

The proposed model is implemented in MATLAB 2014a with the YALMIP toolbox [39] and solved by an academic version of a commercial optimization solver, CPLEX. The optimal operating cost of the system is $53,736.75 with the approval of four out of six maintenance requests. At the optimal solution, few maintenance tasks are divided into partial time frames, as shown in Table VI. For completeness of the study, the values of binary variables $z$ and $u$ at the optimal solution are presented in Table V. Three generators are committed fully in the planning period, and other three are committed partially to meet the hourly power demand.

TABLE V - THE OPTIMAL SOLUTION (SECTION 5.1)
5.2. Model with no partial maintenance

Although the results in Section 5.1 show extra financial savings with the adaptation of the partial maintenance phenomenon, partial maintenance is not generally observed in business. As mentioned in Section 3 when we discussed the constraint (5c), the selection of parameter $\ell_k = 1$ can realize the case of removing the partial maintenance criterion. The same input dataset is used to solve the model with no partial maintenance, and the results are given in Table VI.

5.3. Results of Business-as-usual model

The Business-as-Usual model as adopted by system operators doesn’t consider the tradeoff between the decision on maintenance requests and the total operating costs. Instead, the BaU model has a priority list of maintenance requests based on their submission dates. As long as the first request is $N-1$ reliable for the system, operator approves the request. Then the next request is studied and the decision is made based on the previous approved outage request(s).

In this part of the study, the financial savings of the proposed model in comparison to the BaU is discussed. The priority list is assumed to be Line 31-18-7-38. The optimal solution is presented in Table X. The operating cost of each model discussed in Section 5.1, 5.2, and 5.3 are illustrated in Fig.2.

![Figure 2. All solutions presented in Table VI](image)
OPERATING COSTS WITH VARIOUS SELECTION OF $N_{\text{approve}}$

<table>
<thead>
<tr>
<th>$N_{\text{approve}}$</th>
<th>ORIGINAL MODEL</th>
<th>MODEL w/ NO PARTIAL MAINTANANCE</th>
<th>BaU MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Operating Cost ($)</td>
<td>Approved Line ID(s)</td>
<td>Number of Partial Maintenance</td>
</tr>
<tr>
<td>0</td>
<td>53,781.71</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>53,756.47</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>53,753.67</td>
<td>7, 18</td>
<td>3 – 2</td>
</tr>
<tr>
<td>3</td>
<td>53,736.75</td>
<td>7, 18, 31</td>
<td>2 – 2 – 3</td>
</tr>
<tr>
<td>4</td>
<td>53,736.75</td>
<td>7, 18, 31, 38</td>
<td>2 – 2 – 1 – 3</td>
</tr>
<tr>
<td>5</td>
<td>Infeasible</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>6</td>
<td>Infeasible</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

6. Conclusion

A novel model is developed to centralize the maintenance scheduling process at the system operator level. Central scheduling benefits the operator in terms of production cost savings. Cost savings are achieved byoptimally scheduling the transmission requests at the time their inexistence leads to a lower production cost for the system. The results promise production cost savings by shifting a maintenance time window to one that is optimal for both operator and transmission owner. The model is also developed to be flexible having different type of maintenance requests such as a block or partial maintenance. The model is capable of splitting maintenance duration into several smaller time windows if it leads to a better solution for the system. Moreover, the model includes constraints of SCUC problem and the $N-I$ reliability criterion in order to satisfy secure and reliable system conditions.

The possible versions of the model and their comparisons with the Business-As-Usual model are discussed. Around 1% production cost savings are observed when the system is enforced by the $N-I$ reliability criterion. Moreover, around 4% cost savings are achieved when the system is relaxed by dropping the $N-1$ criterion. The observations on the results show that the model always yields a better solution than any other scheduling method in business.

Appendix
Sets
\( \Phi \): Set of all buses
\( \Phi^G \): Set of buses connected to a generator, a subset of \( \Phi \)
\( \Omega \): Set of all transmission lines (branches)
\( \Psi \): Set of branches pending maintenance, a subset of \( \Omega \)
\( \Gamma \): Set of all generators
\( \chi \): Set of system states including steady state and loss of either a generator or a non-radial transmission line

Indices
\( t \): Period index
\( i, j \): Bus indices for sets \( \Phi \) and \( \Phi^G \)
\( k \): Branch index for sets \( \Omega \) and \( \Psi \)
\( g \): Generator index for set \( \Gamma \)
\( c \): State index, 0 for steady state, rest for contingency state

Parameters
\( Base^{MW} \): Scalar value converting per unit quantity to power
\( C_g \): Marginal generating cost of Generator \( g \)
\( NL_g \): No load cost of Generator \( g \)
\( SU_g \): Startup cost of Generator \( g \)
\( PM_k \): Partial maintenance cost of Branch \( k \)
\( B_k \): Electrical susceptance of Branch \( k \)
\( P_{D_{ij}} \): Real power demand at Bus \( i \) in period \( t \)
\( M \): A big number
\( P_{G_{max}}^g \): Maximum generating capacity of Generator \( g \)
\( P_{G_{min}}^g \): Minimum generating capacity of Generator \( g \)
\( R_{U}^g \): Maximum ramp up rate of Generator \( g \)
\( R_{D}^g \): Maximum ramp down rate of Generator \( g \)
\( R_{UC}^g \): Maximum ramp up rate of Generator \( g \) in contingency
\( R_{DC}^g \): Maximum ramp down rate of Generator \( g \) in contingency
\( P_{max}^k \): Maximum power flow limit on Branch \( k \)
\( T \): Number of planning periods
\( d_k \): Duration to complete maintenance for Branch \( k \)
\( n_k \): Minimum duration of each partial maintenance
\( \ell_k \): Maximum number of times Branch \( k \) could be on partial maintenance
\( ref \): Reference bus id
\( p_g \): Minimum up time for Generator \( g \)
\( w_g \): Minimum down time for Generator \( g \)

Variables
\( P_{G_{g,t,c}} \): Power generation by Generator \( g \) in period \( t \) at state \( c \)
\( \theta_{i,t,c} \): Voltage angle in radians at Bus \( i \) in period \( t \) at state \( c \)
\( P_{kij,t,c} \): Power flow on Branch \( k \) from Bus \( i \) to Bus \( j \) in period \( t \) at state \( c \)
\( z_{k,t} \): Indicating status of Branch \( k \) in period \( t \)
\( m_{k,t} \): Indicating start of maintenance on Branch \( k \) in period \( t \\n\( a_{t} \): Indicating approval of maintenance request for Branch \( k \\n\( u_{g,t} \): Indicating status of Generator \( g \) in period \( t \\n\( s_{g,t} \): Indicating startup decision of Generator \( g \) in period \( t \\n\( h_{g,t} \): Indicating shutdown decision of Generator \( g \) in period \( t \\n
7. References


