Holomorphic Embedding
Load Flow Method

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Backgound

• Holomorphic function
  • A holomorphic function is a complex-valued function of one or more complex variables that is complex differentiable in a neighborhood of every point in its domain.
  • If the derivative of $f$ at a point $z_0$:
    $$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}.$$ 
    exist, we say that $f$ is complex-differentiable at the point $z_0$.
    If $f$ is complex differentiable at every point $z_0$ in an open set $U$, we say that $f$ is holomorphic on $U$. 
• A continuous function which is not holomorphic is the \textit{complex conjugate}
• The right hand side is left with constant-injection and constant-power components.

\[ \sum_{k} Y_{ik} V_{k} = I_{i}^{\text{load}} + \frac{S_{i}^{*}}{V_{i}^{*}} \]

the idea is, if we introduce a variable s, \( V=V(s) \), and

• At \( s=s_{1} \), \[ \sum_{k} Y_{ik} V_{k} = I_{i}^{\text{load}} + \frac{S_{i}^{s}}{V_{i}^{*}} \] holds.

• At \( s=s_{0} \), problem is relatively easy to solve.
• \( V=V(s) \) is Holomorphic

Then we can get form of \( V(s) \) on \( s=s_{0} \), and get value of \( V(s_{1}) \)
• Obvious choice is:

\[ \sum_k Y_{ik} V_k(s) = s I_i^{\text{load}} + \frac{s S_i^*}{V_i^*(s^*)} \]

• Now, V become a function of s. \( V_i^*(s^*) \) is used, not \( V_i^*(s^-) \), to make the function Holomorphic.

• The equation is obviously solvable at s=0
\[ \sum_{k} Y_{ik} V_{k}(s) = sI_{i}^{\text{load}} + \frac{sS_{i}^{*}}{V_{i}^{*}(s^{*})} \]

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- If we claim that \( \bar{V}_{i}(s) \) and \( V_{i}(s) \) are independent, they are all holomorphic.
- When \( \bar{V}_{i}(s) = V_{i}^{*}(s^{*}) \), the solution is physical solution.
Since $U(s)$ is holomorphic, consider the power series expansion about $s=0$.

$$V_i(s) = \sum_{n=0}^{\infty} c_i[n] s^n$$

$$1/V_i(s) = \sum_{n=0}^{\infty} d_i[n] s^n$$

Make use of $\bar{V}_i(s) = V_i^*(s^*)$, 

$$\sum_k Y_{ik} \sum_{n=0}^{\infty} c_k[n] s^n = s I_i^{\text{load}} + s S_i^* \sum_{n=0}^{\infty} d_i^*[n] s^n$$
• After get coefficients in $V_i(s) = \sum_{n=0}^{\infty} c_i[n] s^n$, Padé Approximation is need to get $V_i(1)$

• Padé Approximation:
  • In mathematics a Padé approximant is the "best" approximation of a function by a rational function of given order – under this technique, the approximant's power series agrees with the power series of the function it is approximating.
  • The Padé approximant often gives better approximation of the function than truncating its Taylor series, and it may still work where the Taylor series does not converge.
• Why we can use $\overline{V}_i(s) = V_i^*(s^*)$?

• There are multiple solution at $s=0$. Obviously, at most one of them are physical. If the physical solution exists on $s=0$, we generate the polynomial expansion of $V$ based on this physical solution. From this polynomial form, we can guarantee that $V(s=1)$ is physical, which means $\overline{V}_i(s) = V_i^*(s^*)$ always hold.
performance

• In real-world large transmission network of about 3000 nodes, the HELM algorithm solves Power Flow Equations in 10 to 20 ms