Seeking Global Optimum of AC OPF

Part I

Progress Presentation
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Outline

- History of OPF
- Survey of approaches to solve AC OPF
  - Unconstrained Non-linear Programs
  - Constrained Non-linear Programs
- Feasible Region of AC OPF
- Problems of SDP
- Closing the duality gap
History of OPF

- First digital solution of PF Problem
  - Ward, 1956
- First OPF Formulation - Carpentier (1962)
- Non-linear ---- Non-convex Problem
- Sparsity techniques (Stott 1974)

- Solvers:
  - No guarantee for the global optimum so far.
ACOPF Problem Structure

- AC Power flow equations
- Generator real and reactive power constraints
- Bus voltage magnitude constraints
- Bus voltage angle difference constraints
- Thermal limit of transmission lines

Computational Approaches

- POLAR
- RECTANGULAR
- CURRENT-VOLTAGE
AC OPF Problem – Polar Formulation

\[
\min_{\theta, |V|, P, Q} \sum_{i \in N} f_i (P_{Gi}, Q_{Gi})
\]

\[
|V_i| \sum_{n \in N} |V_n| (G_{in} \cos \theta_{in} + B_{in} \sin \theta_{in}) - P_{Gi} + P_{Di} = 0
\]

\[
|V_i| \sum_{n \in N} |V_n| (G_{in} \sin \theta_{in} - B_{in} \cos \theta_{in}) - Q_{Gi} + Q_{Di} = 0
\]

Cost Function

(1) Real Power Flow, \( \forall i \in Nodes \)

(2) Reactive Power Flow, \( \forall i \in Nodes \)

(3) Generator Real Power Limits, \( \forall i \in Nodes \)

(4) Generator Reactive Power Limits, \( \forall i \in Nodes \)

(5) Node Voltage Magnitude Limits, \( \forall i \in Nodes \)

(6) Maximum Phase Angle Difference, \( \forall i \in Interconnections \)

(7) Line Thermal Limits, \( \forall l \in Lines \)
AC OPF Problem – Rectangular Formulation

\[
\begin{align*}
\min_{V_{im},V_{re},P,Q} \sum_{i \in N} f_i (P_{Gi}, Q_{Gi}) \\
V_{re,i} \sum_{n \in N} (G_{in}V_{re,n} - B_{in}V_{im,n}) \\
+ V_{im,i} \sum_{n \in N} (G_{in}V_{im,n} + B_{in}V_{re,n}) - P_{Gi} + P_{Di} &= 0 \\
V_{im,i} \sum_{n \in N} (G_{in}V_{re,n} - B_{in}V_{im,n}) \\
- V_{re,i} \sum_{n \in N} (G_{in}V_{im,n} + B_{in}V_{re,n}) - Q_{Gi} + Q_{Di} &= 0 \\
P_{Gi}^{min} &\leq P_{Gi} \leq P_{Gi}^{max} \\
Q_{Gi}^{min} &\leq Q_{Gi} \leq Q_{Gi}^{max} \\
V_i^{min} \leq &\sqrt{V_{re,i}^2 + V_{im,i}^2} \leq V_i^{max} \\
\theta_{in}^{min} \leq &\text{arctan} (V_{im,i}/V_{re,i}) - \text{arctan} (V_{im,n}/V_{re,n}) \leq \theta_{in}^{max} \\
\sqrt{P_{inl}^2 + Q_{inl}^2} &\leq s_{inl}^{max}
\end{align*}
\]

Cost Function

(1) Real Power Flow, \(\forall i \in Nodes\)

(2) Reactive Power Flow, \(\forall i \in Nodes\)

(3) Generator Real Power Limits, \(\forall i \in Nodes\)

(4) Generator Reactive Power Limits, \(\forall i \in Nodes\)

(5) Node Voltage Magnitude Limits, \(\forall i \in Nodes\)

(6) Maximum Phase Angle Difference \(\forall in \in Interconnections\)

(7) Line Thermal Limits \(\forall l \in Lines\)
Unconstrained Nonlinear Optimization

- Minimize a non-linear function $f(x)$

$$f^* = \inf \{ f(x) \mid x \in X \}$$

- Solution Process

  Step 1. Choose a function to optimize; set $k = 0$. Choose an initial point: $x_0$.
  Step 2. Choose a search direction: $d_k$.
  Step 3. Choose step size $s_k$ where $s_k$ is a positive scalar and calculate a new point: $x_{k+1} = x_k + s_k d_k$.
  Step 4. Test for stopping: If $x_{k+1}$ satisfies the convergence criteria or exceeds the time allotted, then set $K = k+1$ and stop. Otherwise, if $x_{k+1}$ does not satisfy the convergence criteria, then set $k = k + 1$ and go to Step 2.
Methods

- Gauss – Seidel
- Steepest Descent
- Conjugate Gradient
- Newton

Challenges

- Zigzagging related to step size
- Inverse of A is numerically unstable.
Constrained Nonlinear Optimization

- Minimize a function $f(x)$

$$f^* = \inf \{ f(x) \mid g(x) \leq 0, h(x) = 0, x \in X \} \quad (P)$$

- Karush-Kuhn-Tucker (KKT) Conditions

$$\nabla f(x) + \lambda^T \nabla g(x) = 0,$$
$$\lambda \geq 0,$$
$$\lambda^T g(x) = 0,$$
$$h(x) = 0.$$

- Necessary for local optima, but not sufficient for global optimum in non-convex set
Lagrangian – Augmented Lagrangian

- **Lagrangian Dual:**
  \[ L(x, \lambda) = f(x) + \lambda^T g(x) \]

- **Augmented**
  \[ \Lambda(g(x), \lambda, \mu) = \lambda^T g(x) + \mu^T g(x)^2 \]

Step 1. Choose \( \lambda_0, \mu_0 \) in iteration \( k = 0 \).

Step 2. Find \( x_{k+1} = \text{argmin}_x \{ f(x_k) + \lambda_k^T g(x_k) + \mu_k^T g(x_k) \} \).

Step 3. Update \( \mu_{k+1} > \mu_k \) and \( \lambda_{k+1} = \lambda_k + \mu_k^T g(x_k) \).

Step 4. If convergence criteria is met, then stop; else go to Step 2.
Barrier / Interior Point Method

- Initial point is either feasible or infeasible point

Logarithmic Barrier: \( \Lambda(g(x), \lambda, \mu) = \mu^T \log(-g(x)) \).

\[
\min \ f(x) - \mu_k^T \log(-g(x))
\]

where

\( \mu_k > 0 \), and

\[
\frac{\partial (\mu_k^T \log(-g_i(x_k)))}{\partial x_k} = \mu_k^T \nabla g_i(x_k) / g_i(x_k).
\]
## Conic and Semi Definite Programming

**Primal**

\[
\begin{align*}
\min_x & \quad c^T x \\
\text{subject to} & \quad Ax = b \\
& \quad x \geq 0
\end{align*}
\]

**SDP Relaxation**

\[
\begin{align*}
\min_x & \quad c^T x \\
\text{subject to} & \quad Ax + b \in K
\end{align*}
\]

- K is a convex cone
- Avoid local minima by relaxing obj. or domain
Feasible Region of OPF

- Hiskens and Davy, 2001, Exploring the Power Flow Solution Space Boundary
Exploration of Feasible Region

1) Is ACOPF problem nearly convex?
   - No strong evidence. Problem has some convex properties, but too many irregularities in reactive power.

2) Is region convex around the global optimum?
   - No feasible convex combinations of two feasible points.

3) Is region very dynamic or flat?
   - Globally flat, but locally dynamic.

4) What is pair-wise relationship between variable values and cost?
   - Many variables $\rightarrow$ smooth quadratic behavior
   - Other $\rightarrow$ Highly irregular points
SDP Relaxation of AC OPF

Implementations
- YALMIP
- SeDuMi solver
- Mac and PC Compatible

Accepts MATPOWER case file as an input
- Voltage Magnitude Limits for each bus
- Real and Reactive Power Demand at each bus
- Real and Reactive Power Limits of each generator
- Polynomial cost of each generator
- Long term thermal limit of each branch
- Resistance, Reactance, and line susceptance of each branch
- Branch Status – In service or out of service
Add-on Features

- Other than SDP formulation adopted from (Lavaei, Low)

1. Reference Angle Constraint
2. Thermal line limit at each end
3. Multiple generator at a bus
4. Parallel Transmission Lines
Additional AC OPF Formulations

- Rectangular Current Voltage Formulation (O’Neill, Castillo, 2013)
  - Next week
- Convex Quadratic Programming (Hassan, 2013)
  - In polar form
    - Convex cosine function
    - Polyhedral sine function
    - Convex voltage magnitude
    - Tight bound by real power loss constraint
- Applied to OPF, Optimal Transmission Switching, Capacitor Placement
Sufficient Condition for Global Optimality

- SDP Relaxation is a convex problem.
- Only the global optimal ($x^*$) can satisfy the KKT conditions in a convex set.